Mid-Semester Examination
I Semester, 2008-2009

B. Math III Year
Complex Analysis

1. Find a Mobius transformation S from $\mathbb{R} \cup \{\infty\} \to \{z : |z| = 1\}$ which is surjective. Find the image under this transformation of $\{z : \mathrm{Im}(z) > 0\}$. [25]

2. Find the harmonic conjugate of $u(x, y) = \sin x \cosh y$ vanishing at $(1, 0)$. [15]

3. Give an example of a region Ω and a function f in $H(\Omega)$ such that there is no power series convergent at all points of Ω whose sum is $f(z)$. [15]

4. If Ω is a region and f^2 and f^- are analytic in Ω show that f is necessarily a constant on $\dot{\Omega}$. [15]

5. If $\gamma : [0, 1] \to \mathbb{C}$ is continuously differentiable show that $\int_\gamma \frac{1}{\zeta - z} d\zeta \to 0$ as $z \to \infty$. [10]

6. Find the nature of singularity of the following functions at 0 :

a) $\frac{Log(1+z)}{z^2}$

b) $\frac{1}{1-e^z}$

c) $z^2 \sin(\frac{1}{z})$ [5 + 5 + 5]

7. If f is a given entire function, find all entire functions g such that $|g(z)| \leq |f(z)|^2$ for all $z \in \mathbb{C}$. [15]